

Yukawa Textures in Heterotic M-Theory

R. Arnowitt and B. Dutta

*Department of Physics, Texas A&M University
College Station, TX 77843-4242, USA*

We examine the structure of the Yukawa couplings in the 11 dimensional Horava-Witten M-theory based on non-standard embeddings. We find that the CKM and quark mass hierarchies can be explained in M Theory without introducing undue fine tuning. A phenomenological example is presented satisfying all CKM and quark mass data requiring the 5-branes cluster near the second orbifold plane, and that the instanton charges of the physical orbifold plane vanish. The latter condition is explicitly realized on a Calabi-Yau manifold with del Pezzo base dP_7 .

In Horava-Witten heterotic M-theory¹ with “non-standard” embeddings space has an 11 dimensional orbifold structure of the form (to lowest order) $M_4 \times X \times S^1/Z_2$ where M_4 is Minkowski space, X is a 6 dimensional (6D) Calabi-Yau space, and $-\pi\rho \leq x^{11} \leq \pi\rho$. There are two orbifold 10D manifolds $M_4 \times X$ at the Z_2 fixed points at $x^{11} = 0$ and $x^{11} = \pi\rho$, each with an a priori E_8 gauge symmetry. Physical matter lives on the $x^{11} = 0$ orbifold plane and only gravity lives in the bulk.

In addition there can be a set of 5-branes in the bulk at points $0 < x_n < \pi\rho$, $n = 1 \dots N$ each spanning M_4 (to preserve Lorentz invariance) and wrapped on a holomorphic curve in X (to preserve $N=1$ supersymmetry). The existence of these 5-branes allows one to satisfy the cohomological constraints with E_8 on the $x^{11} = 0$ plane breaking to $G \times H$ where G is the structure group of the Calabi-Yau manifold and H is the physical grand unification group. We consider here the case $G = SU(5)$, and hence $H = SU(5)$.

Recently, three generation models with a Wilson line breaking $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ have been constructed in the M-theory frame work using torus fibered Calabi-Yau manifolds (with two sections)². Also, the general structure of the Kahler metric of the matter field has been examined³. We find that this structure can lead to Yukawa textures with all CKM and quark mass data in agreement with experiment. We have also shown that a three generation model with a Wilson line (to break $SU(5)$ to the standard model) and which explains the hierarchies in the matrix elements of the Yukawa sector can be constructed on a Calabi-Yau manifold with del-Pezzo base dP_7 ⁴.

The bose part of the 11 dimensional gravity multiplet consists of the metric tensor g_{IJ} , the antisymmetric 3-form C_{IJK} and its field strength $G_{IJKL} = 24\partial_{[I}C_{JKL]}$. ($I, J, K, L = 1 \dots 11$). The G_{IJKL} obey field equations $D_I G^{IJKL} = 0$ and Bianchi

identities

$$\begin{aligned}
(dG)_{11RSTU} &= 4\sqrt{2}\pi\left(\frac{\kappa}{4\pi}\right)^{2/3}[J^0\delta(x^{11}) \\
&+ J^{N+1}\delta(x^{11} - \pi\rho) + \frac{1}{2}\sum_{n=1}^N J^n(\delta(x^{11} - x_n) \\
&+ \delta(x^{11} + x_n))]_{RSTU}
\end{aligned} \tag{1}$$

Here $(\kappa^{2/9})$ is the 11 dimensional Planck scale, and J^n , $n = 0, 1, \dots, N+1$ are sources from orbifold planes and the N 5-branes. These equations can be solved perturbatively in powers of $(\kappa^{2/3})^3$. The effective 4D theory is then given by a Kahler potential $K = Z_{IJ}\bar{C}^I C^J$, Yukawa couplings Y_{IJK} for the matter fields C^I and gauge functions from the physical orbifold plane $x^{11}=0$. To first order, Z_{IJ} takes the form³.

$$Z_{IJ} = e^{-K_T/3} [G_{IJ} - \frac{\epsilon}{2V} \tilde{\Gamma}_{IJ}^i \Sigma_0^{N+1} (1 - z_n)^2 \beta_i^{(n)}] \tag{2}$$

$\epsilon = (\kappa/4\pi)^{2/3} 2\pi^2 \rho / V^{2/3}$ is the expansion parameter. V is the Calabi-Yau volume, G_{IJ} , $\tilde{\Gamma}_{IJ}^i$ and Y_{IJK} can be expressed in terms of integrals over the Calabi-Yau manifold, and K_T is the Kahler potential for the moduli.

The second term of Eq.(2) is a small correction in order to make the perturbation analysis to work. A priori one expects G_{IJ} , $\tilde{\Gamma}_{IJ}^i$ and Y_{IJK} to be of $O(1)$, and the parameter ϵ is not too small. However, the second term will be small if $\beta_i^{(0)}$ vanishes and if the 5-branes cluster near the distant orbifold plane i.e. $d_n \equiv 1 - z_n$ is small, where $z_n = x_n/\pi\rho$. In the following we will assume then that

$$\beta_i^{(0)} = 0; \quad d_n = 1 - z_n \cong 0.1 \tag{3}$$

The condition $\beta_i^{(0)} = 0$ is quite non trivial, but it is possible to show that a three generation model of a torus fibered Calabi-Yau manifold with Wilson like breaking $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ with del -Pezzo base dP_7 has this property⁴.

Since d_n is small, we will assume that the ϵ term of Eq.(2) constitute the third generation contributions to the Kahler metric. A simple phenomenological example for the u and d quark contributions with these properties (and containing the maximum numbers of zeros) is ($f_T \equiv \exp(-K_T/3)$):

$$\begin{aligned}
Z^u &= f_T \begin{pmatrix} 1 & 0.345 & 0 \\ 0.345 & 0.132 & 0.639d^2 \\ 0 & 0.639d^2 & 0.333d^2 \end{pmatrix}; \\
Z^d &= f_T \begin{pmatrix} 1 & 0.821 & 0 \\ 0.821 & 0.887 & 0 \\ 0 & 0 & 0.276 \end{pmatrix}.
\end{aligned} \tag{4}$$

with Yukawa matrices $\text{diag} Y^u = (0.0765, 0.536, 0.585 \exp[\pi i/2])$ and $\text{diag} Y^d = (0.849, 0.11, 1.3)$.

To obtain the physical Yukawa matrices, one must first diagonalize the Kahler metric and then rescale it to unity. Then using the renormalization group equations,

one can generate the CKM matrix, and the quark masses. The results are given in the following table:

Quantity	Th. Value	Exp. Value ⁵
$m_t(\text{pole})$	170.5	175 ± 5
$m_c(m_c)$	1.36	1.1-1.4
$m_u(1 \text{ GeV})$	0.0032	0.002-0.008
$m_b(m_b)$	4.13	4.1-4.5
$m_s(1 \text{ GeV})$	0.110	0.093-0.125 ⁶
$m_d(1 \text{ GeV})$	0.0055	0.005-0.015
V_{us}	0.22	0.217-0.224
V_{cb}	0.036	0.0381 ± 0.0021 ⁷
V_{ub}	0.0018	0.0018-0.0045
V_{td}	0.006	0.004-0.013

and $\sin 2\beta = 0.31$ and $\sin \gamma = 0.97$. The agreement with experiment is quite good. The quark mass ratios for the first two generations are given as $m_u/m_d = 0.582$ and $m_s/m_d = 20.0$. These values are in good agreement with Leutwyler evaluations⁸ 0.553 ± 0.043 and 18.9 ± 0.8 .

Though the entries in $Z^{u,d}$ and $Y^{u,d}$ are chosen to obtain a precise fit to the experimental results, as shown in ⁴, the quark mass hierarchies arise naturally from a Kahler metric of the type of Eq. (4). Similarly the smallness of the off diagonal V_{CKM} matrix elements also occur naturally as a consequence of the above model. Thus it is possible for M-theory to generate the Yukawa hierarchies without any undue fine tuning and without introducing ad hoc very small off diagonal entries.

M theory with non-standard embeddings is a new possibility of encoding the Yukawa hierarchies in the Kahler metric. This can happen naturally if the 5-branes cluster near the hidden orbifold plane ($d_n \equiv 1 - z_n \simeq 0.1$) and the instanton charges of the physical plane vanish ($\beta_i^{(0)} = 0$).

This work was supported in part by NSF grant no. PHY-9722090.

References

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